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# Transition from regular to Mach reflection of shock waves Part 2. The steady-flow criterion

By H. G. HORNUNG†
Department of Physics, Faculty of Science, Australian National University, P.O. Box 4, Canberra, ACT 2600

AND M. L. ROBINSON
Weapons Systems Research Laboratory, Defence Research Centre, Salisbury, South Australia

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It is shown experimentally that, in steady flow, transition to Mach reflection occurs at the von Neumann condition in the strong shock range (Mach numbers from 2·8 to 5). This criterion applies with both increasing and decreasing shock angle, so that the hysteresis effect predicted by Hornung, Oertel & Sandeman (1979) could not be observed. However, evidence of the effect is shown to be displayed in an unsteady experiment of Henderson & Lozzi (1979).

## 1. Introduction

In this paper we consider the transition to Mach reflection in the case of a uniform strong shock wave in two-dimensional steady flow of a thermally and calorically perfect gas. Viscous effects at the reflecting wall are avoided by replacing it with a plane of symmetry.

Figure 1 presents definition sketches of the configurations that occur. In the regular reflection (figure 1*a*) a supersonic stream encounters an oblique shock at an angle $\alpha$ and is deflected towards the reflecting wall. The reflected shock provides the means of turning the flow back to its original direction parallel to the wall. As $\alpha$ is increased, there comes a point $\alpha = \alpha^*$ at which the configuration changes to the Mach reflection shown in figure 1(*b*). The length $s$ of the nearly normal shock or 'Mach stem' $S$ is controlled by the geometry of the upper boundary of region 2. This boundary cannot influence the reflection point in the regular-reflection configuration (figure 1*a*) because the flow is everywhere supersonic. In the Mach reflection, however, the flow after the Mach stem is subsonic, so that upstream influence is possible in region 4.

The symmetrical configuration to be used in the experiment is shown in figure 2. This configuration is almost essential, because the viscous boundary layer on the wall is often separated by the shock, causing confusing complications. This configuration cannot, of course, avoid the viscous growth of the shear layer between regions 3 and 4, which was considered by Skews (1971) and Sternberg (1959). Since this is only present after the transition to Mach reflection has occurred, it is not expected to influence the failure of regular reflection, but might affect the reverse transition from Mach reflection to regular reflection.

To explain the mechanism by which the scale of the upper boundary of region 2 controls the scale of the Mach reflection, consider the top half of figure 2. The

† Present address: DFVLR-AVA, 34 Göttingen, Bunsenstr. 10, W. Germany.

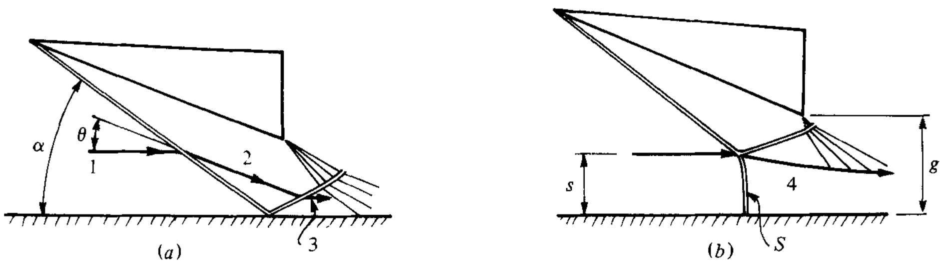



FIGURE 1. Defining parameters of (*a*) regular and (*b*) Mach reflection configurations. Secondary reflections of expansion waves have been omitted for clarity.

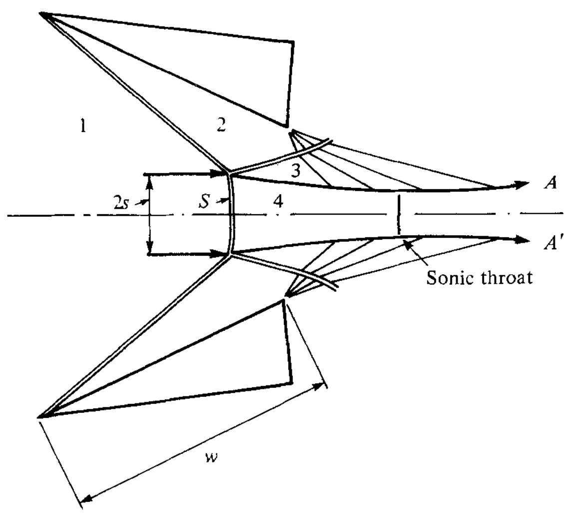



FIGURE 2. Symmetrical arrangement for avoiding viscous effects at the wall. The flow in region 4 is subsonic up to the throat formed by the streamlines $A$ and $A'$. Secondary reflections of expansion waves have been omitted for clarity.

expansion fan from the trailing edge of the shock-generating wedge eventually strikes the streamline $A$ that passed through the triple point. This causes the pressure to drop in the streamwise direction in region 4, thus accelerating the flow, eventually, to supersonic conditions again. As a result, the cross-sectional area of the stream tube between the two triple-point streamlines $A$ and $A'$ decreases initially to a minimum at a sonic throat, and then increases again in the region of accelerating supersonic flow. A subsonic pocket is therefore formed in an otherwise supersonic flow. The size and shape of this pocket are controlled by the distance between the Mach stem and the sonic throat, which in turn depends on the geometry of the upper boundary of region 2, and the distance $g$ between the trailing edge of the wedge and the symmetry plane. This may be expressed formally as

$$\frac{s}{w} = f_1\left(\frac{g}{w}, \theta, M_1, \gamma\right) \tag{1}$$

for inviscid flow, where $M_1$ and $\gamma$ are the Mach number in region 1 and the ratio of specific heats of the gas, and the geometric variables $s$, $w$, $g$ and $\theta$ are defined in figures 1 and 2. Since the shock angle is determined by a relation of the form

$$\alpha = \alpha(\theta, M_1, \gamma), \tag{2}$$

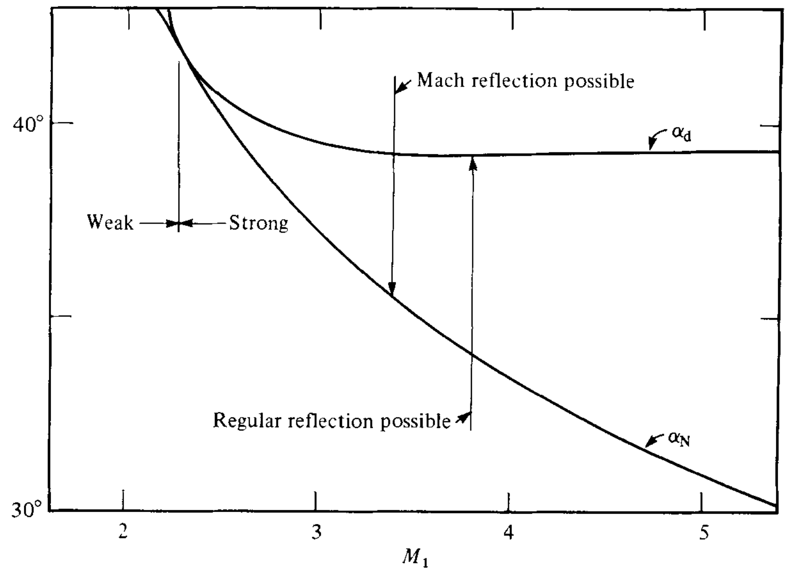



FIGURE 3. The von Neumann and detachment criteria as functions of Mach number for specific-heat ratio $\gamma = \frac{7}{5}$.

(1) may be rewritten as

$$\frac{s}{w} = f_2\left(\frac{g}{w}, \alpha, M_1, \gamma\right). \tag{3}$$

This assumes, of course, that the pressure downstream of the model is sufficiently low, and that $g$ is sufficiently large, to permit supersonic flow to be established at the downstream end of the stream tube between $A$ and $A'$. Situations where this is not the case are considered by Henderson & Lozzi (1979), who observe an interesting hysteresis effect related to it. They also consider steady-flow Mach reflections with non-uniform (curved) shocks, which can lead to a situation where the streamlines $A$ and $A'$ initially diverge.

According to theory (see e.g. Courant & Friedrichs 1948) Mach reflection is not possible when $\alpha < \alpha_N$, where $\alpha_N$ (see figure 3) is that value of $\alpha$ for which the pressure after the regularly reflected shock (region 3) is equal to the pressure reached from region 1 via a normal shock. Also, regular reflection is not theoretically possible for $\alpha > \alpha_d$, where $\alpha_d$ marks the point at which the regularly reflected shock is at the condition of maximum streamline deflection. In the region

$$\alpha_N < \alpha < \alpha_d, \tag{4}$$

both configurations are possible. Figure 3 illustrates the ranges of possible regular and Mach reflections as functions of $M_1$. It also shows the boundary between 'strong' and 'weak' shocks at the tangent point of $\alpha_d$ and $\alpha_N$.

Most of the early experiments on strong Mach reflection have been done in pseudosteady flow, and support the detachment criterion $\alpha^* = \alpha_d$. Steady-flow experiments were restricted to low Mach numbers, where the difference between $\alpha_d$ and $\alpha_N$ is small. It is not surprising therefore that textbooks such as Liepmann & Roshko (1957), Landau & Lifshitz (1959), Becker (1966) and Whitham (1974) give $\alpha^* = \alpha_d$ as the transition criterion for pseudosteady and steady flow.

It was through the work of Henderson & Lozzi (1975) that the von Neumann criterion $\alpha^* = \alpha_N$ came back. Though their steady-flow results extended only to $M_1 = 3$, $\alpha^*$ was measured clearly below $\alpha_d$. Hornung & Kychakoff (1977) followed this up with an isolated perfect-gas experiment in argon at $M_1 = 16$, which clearly confirmed $\alpha^* = \alpha_N$ in a range where $\alpha_d - \alpha_N = 9°$. This was later confirmed by Hornung, Oertel & Sandeman (1979), who also showed that the same criterion applies for steady flow of a real gas and gave a physical explanation for the difference between the criteria for the steady and pseudosteady cases, based on the availability of information about a lengthscale at the reflection point. For the steady case it is argued that if, in the range $\alpha_N < \alpha < \alpha_d$, a Mach reflection is temporarily set up by means of a disturbance (such as may occur during tunnel starting) then an information path from the trailing edge of the wedge to the reflection point is set up by virtue of the subsonic pocket following the Mach stem (see figure 2). The Mach reflection configuration is therefore stable in the range where both configurations are possible, and the criterion for transition is $\alpha^* = \alpha_N$.

Hornung *et al.* (1979) proposed an experiment to test this idea with a model in which $\alpha$ can be changed during the run in a steady-flow facility, so as to avoid the large starting disturbances of their own experiments. By varying $\alpha$ through the range $\alpha_N < \alpha < \alpha_d$, both from below and from above, it was suggested that a hysteresis effect might be observed. Note that this should also indicate any effect due to the viscous growth of the shear layer along the triple-point streamline, as this may be expected to depend on the direction of $\alpha$-change.

The aim of the present experiments was firstly to obtain a more complete set of data for steady-flow transition by measuring $\alpha^*$ at four values of $M_1$ in a range ($2{\cdot}8 \leqslant M_1 \leqslant 5$) where $\alpha_d - \alpha_N$ is such that the experiment can conclusively decide between the criteria, and secondly to measure $\alpha^*$ while increasing and while decreasing $\alpha$ through the range $\alpha_N < \alpha < \alpha_d$ during steady conditions, in order to examine the suggestion that a hysteresis effect may occur.

## 2. Experiment

The blowdown wind tunnel S3 of the Weapons Systems Research Laboratory of the Defence Research Centre, Salisbury, South Australia, is the facility in Australia that is most suitable for our experiment. A calibration and a detailed description are given by Robinson (1970) and by Robinson & Landers (1967). The tunnel has a rectangular test section $152 \times 178$ mm.

Air, dried to a water content of 150 p.p.m. by weight, is supplied to the control valve of the tunnel at pressures up to 8 MPa. This air is heated to the required reservoir temperature (up to 370 K) as it flows through the regenerative heater into the settling chamber. The heater stores enough energy prior to the run to maintain the reservoir temperature constant to within $\pm 3$ K for the duration of the run (typically 30 s). The reservoir pressure (2·76 MPa maximum) is controlled to within $\pm 1$ % automatically during the run. The Mach number is determined by interchangeable nozzle blocks designed for $M = 2{\cdot}8$, 3·5, 4·0 and 5·0 in the test diamond centred on the maximum cross-section. The variation of $M$ over the region covered by the model of the present experiment is less than 0·5 % at all four Mach numbers.

The tunnel is equipped with an incidence-change mechanism and a good quality schlieren/shadowgraph system, the windows of which give coverage of the region between the top and bottom surfaces of the test section. The schlieren system is fitted with a 35 mm camera equipped with an automatic expose/film-wind mechanism that allows exposures to be taken at a rate of up to 3 frames per second with the spark light source triggered from the camera. A schlieren image is displayed in the control room on closed circuit television to give the operator the information necessary to make sensible decisions about experimental procedure.

For the purposes of our experiment, the model consisted of a double-wedge arrangement similar to that of figure 2, with a system for adjusting the incidence of the wedges symmetrically using the incidence-change mechanism. This enabled the wedge incidence to be varied continuously during a run at a minimum rate of 1·8° $s^{-1}$, thus giving a resolution for $\theta$ of approximately 0·6° $\mathrm{frame}^{-1}$. The dimension $w$ was 50 mm.

The location of the pivot pins for the wedges was chosen in such a manner that the gap between the trailing edges does not change excessively with wedge angle, and that the aerodynamic forces produce a slight incidence-reducing moment. It was not possible to reduce backlash in the system to less than $\pm 1°$. However, the accuracy of determination of wedge angle from photographs is estimated to be $\pm 0{\cdot}25°$.

Initially, the wedges extended to within 2 mm of the sidewalls in the spanwise direction, provoking an undesirable interaction with the sidewall boundary layer. Subsequently the span of the wedges was reduced to 102 mm, giving 25 mm clearance from the sidewall. Whilst the modification eliminated the undesirable interaction with the sidewall bounday layer, oil-flow studies showed that the flow over one tip of each wedge was affected by blockage in the side support arms. This was subsequently corrected and satisfactory shock waves were generated by the wedges. Because of the finite span of the wedges, however, the shock-interaction pattern is plane only in the central third of the span.

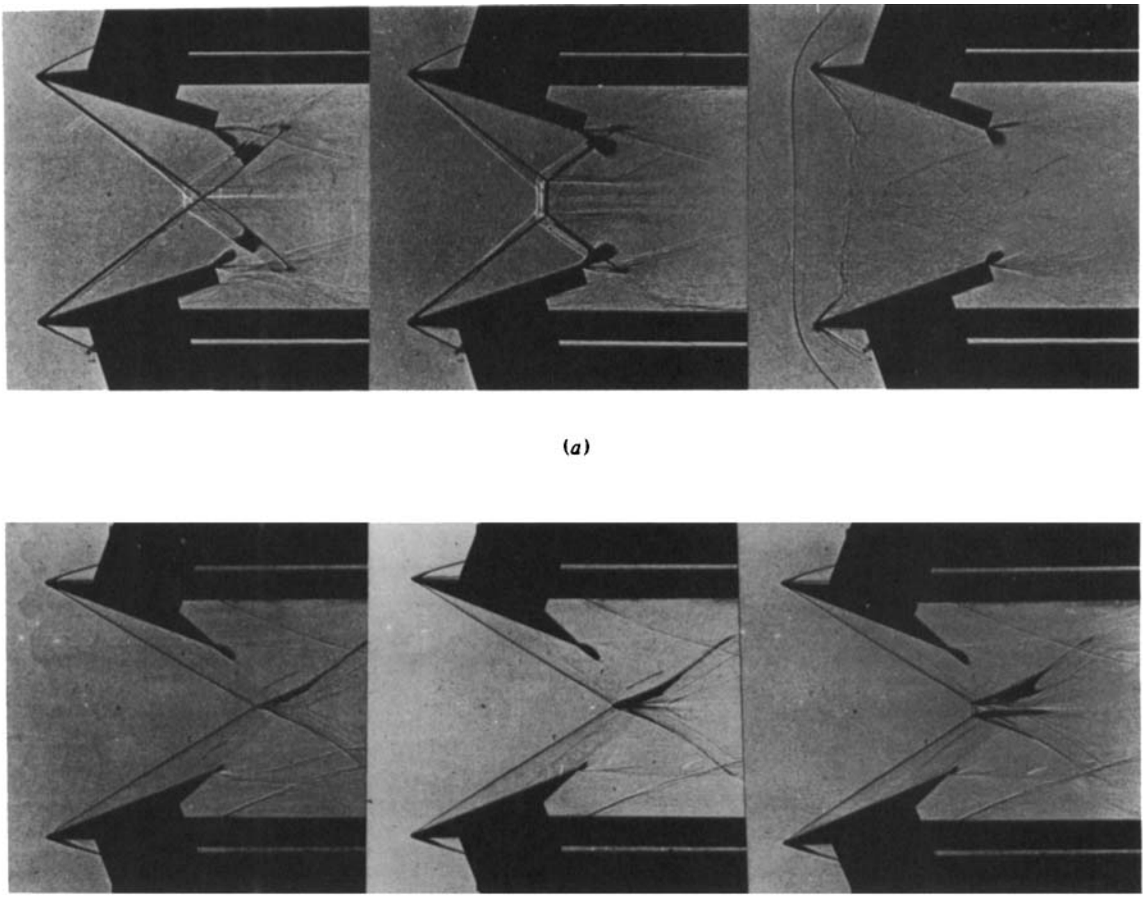
The experimental conditions are given in table 1. These conditions could not be reproduced exactly in repeat runs, but the effect of the run-to-run variation on the present experiment is not significant.

| $M$ | $T_0$ (K) | $p_0$ (MPa) |
|---|---|---|
| 2·84 | 300 | 0·31 |
| 3·49 | 300 | 0·49 |
| 3·98 | 300 | 0·76 |
| 4·96 | 365 | 1·54 |

TABLE 1. Experimental conditions

## 3. Results

The first series of experiments was performed with the model that provides only a narrow spanwise gap between it and the window. It was immediately clear that no hysteresis effect occurred, the transition being independent of the direction of the incidence change. Examples of the shadowgraph pictures taken are presented in figure 4. These show strong evidence of an interaction of the wedge shock with the sidewall (window) boundary layer in the form of a feature slightly upstream of the shock. This feature is a three-dimensional separation and reattachment of the window boundary layer caused by the oblique impingement of the wedge shock. The more sharply defined line may be identified as the wedge shock by comparing its incidence with that calculated from $\theta$, $\gamma$ and $M_1$ (equation (2)). Both in these and later experiments the agreement of the measured and calculated shock angles was excellent. However,



(*a*)

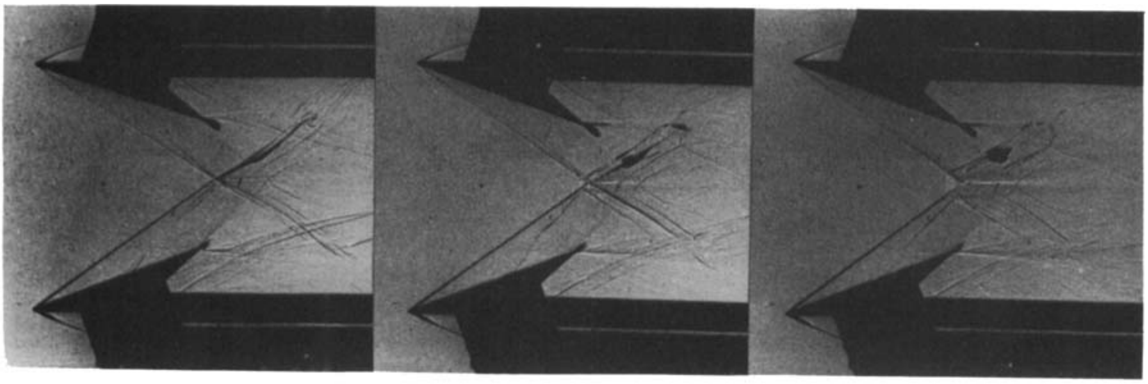
(*b*)

FIGURE 4. Shadowgraph of transition with sidewall interference. Note the shock–boundary-layer interaction upstream of the shock. (*a*) $M_1 = 2{\cdot}84$; (*b*) $M_1 = 4{\cdot}96$. The 'choked' configuration in the third picture of (*a*) occurs also at the other Mach numbers tested for sufficiently high values of $\theta$.

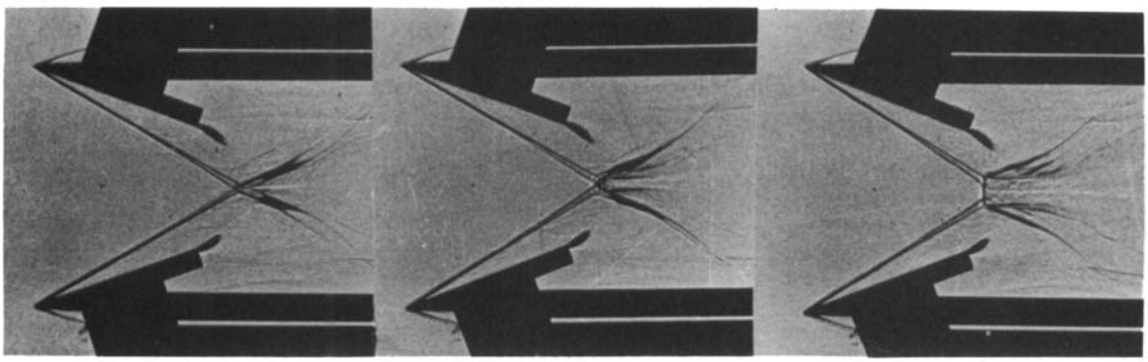
the measured *transition* angle $\alpha^*$ was slightly but significantly smaller than $\alpha_N$ throughout the Mach-number range tested.

The experiments were then repeated with models on which the span was reduced to 102 mm to move the boundary-layer interaction further downstream. Following modifications to the support arms, this was successful in producing well-defined shock fronts, and examples of shadowgraph photographs taken with this arrangement are presented in figure 5 for two values of $M_1$. The shock waves visible just downstream of the wedge shocks originate from the side support arms adjacent to the sidewalls. They have no effect on the wedge–shock interaction.

Again the transition angles were observed to be independent of the direction of incidence change. The values of $\alpha^*$ were determined by plotting the dimensionless Mach stem length $s/w$ against $\alpha$, fitting a straight line to the measurements and extrapolating to $s/w = 0$. Examples of such plots are shown in figure 6 and compared with $\alpha_N$.



(*a*)



(*b*)

FIGURE 5. Shadowgraphs of transition with sidewall interference largely eliminated. (*a*) $M_1 = 3{\cdot}98$; (*b*) $M_1 = 4{\cdot}96$.

The experimental results for $\alpha^*$ are presented in figure 7 in relation to the two curves for $\alpha_N$ and $\alpha_d$ together with data from Henderson & Lozzi (1975). This demonstrates clearly that the von Neumann criterion gives the correct transition angle in steady flow, over the whole Mach-number range. The large separation of the two curves at $M_1 = 5$ ($8{\cdot}5°$) and the excellent agreement of $\alpha^*$ with $\alpha_N$ of our data give more convincing confirmation of this result than Henderson & Lozzi's (1975) data, because of the proximity of $\alpha_N$ and $\alpha_d$ at their lower Mach number.

## 4. Hysteresis of $\alpha^*$

Figure 8 shows a schematic diagram of $s/w$ versus $\alpha$, which illustrates the type of hysteresis that might be expected on the basis of the hypothesis put forward by Hornung *et al.* (1979). As has been pointed out in §3, our experiments, in which $\alpha$ was changed during the run, were not able to confirm this. It must be concluded that either the effect does not occur at all or the disturbances present in our flow were sufficient to cause the regular reflection, which is unstable in $\alpha_N < \alpha < \alpha_d$ according to the argument of Hornung *et al.* (1979), to flip to the Mach reflection (upper curve).

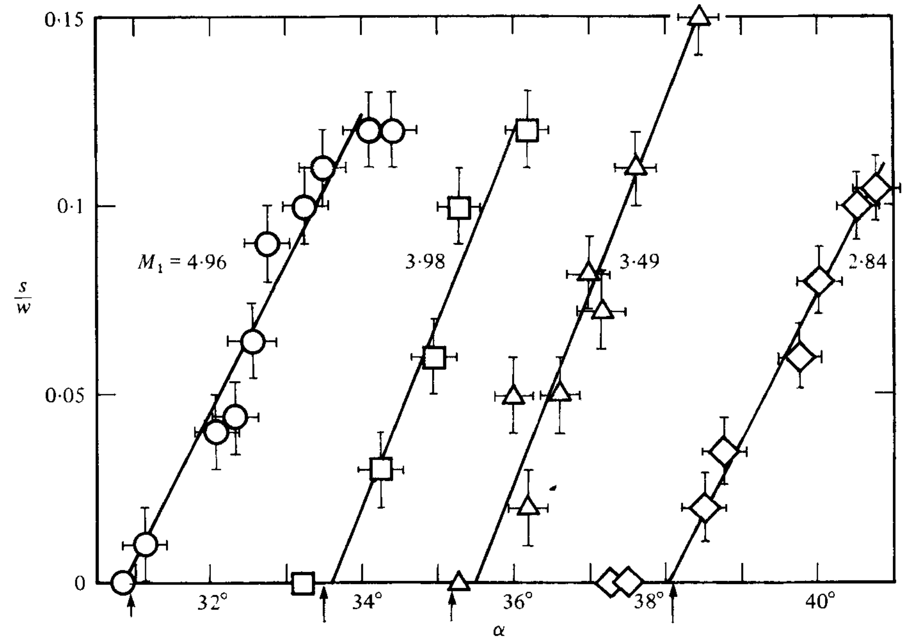



FIGURE 6. Measured dimensionless Mach stem lengths. The straight lines are least-squares fits to the experimental points. The arrows indicate corresponding values of $\alpha_N$.

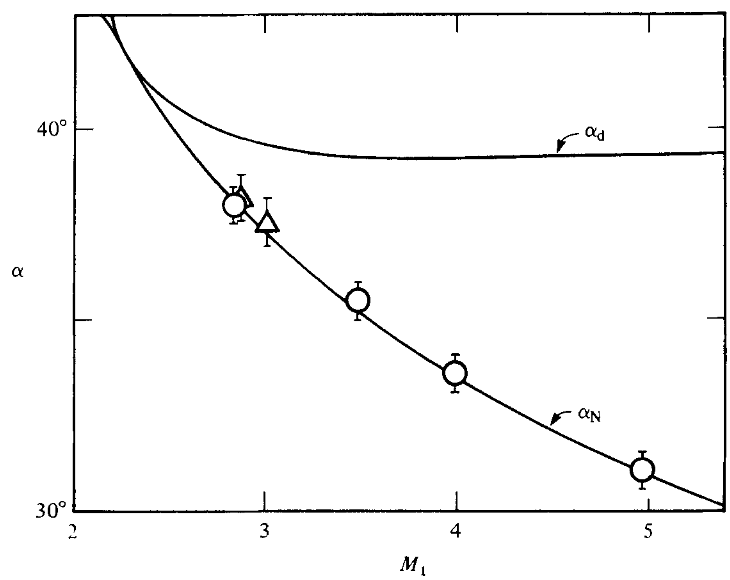



FIGURE 7. Measured transition angle compared with theory. △, Henderson & Lozzi (1975); ○, our data.

It turns out that Henderson & Lozzi (1979) performed an experiment in unsteady flow (with quite different aims) that gives an indication that this hysteresis does in fact take place. In this experiment a plane shock travels through a stationary gas and is reflected from a stationary wedge. The wedge surface is concave, however, so that $\alpha$ decreases as the shock proceeds up the wedge. Two sets of experiments

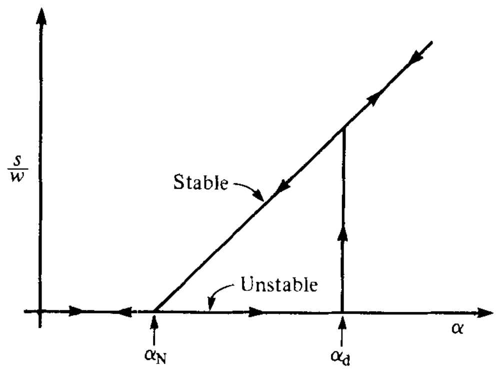



FIGURE 8. Illustrating the hysteresis effect.

(Henderson & Lozzi 1979, figures 5*a*,*b*) were performed with two values of the incidence at the wedge tip, i.e. with two different initial values $\alpha_i$ of $\alpha$, of which one was greater and one just less than $\alpha_d$. For $\alpha_i > \alpha_d$, Mach reflection was observed down to $\alpha = \alpha_N$, below which regular reflection was observed. On the other hand, when $\alpha_N < \alpha_i < \alpha_d$, the reflection configuration was regular, throughout the range, in 9 out of 11 cases. This gives evidence of precisely the type of hysteresis sought, and is thought to occur in Henderson & Lozzi's experiment because of the low level of disturbances inherent in their arrangement.

## 5. Conclusions

It has been demonstrated that, in steady flow, the transition from regular to Mach reflection of strong shock waves occurs at the von Neumann condition and not at the detachment condition. This has been done more convincingly than in previous work by performing experiments in the Mach number range $2{\cdot}8 \leqslant M_1 \leqslant 5$, at the upper end of which the separation of the two criteria is 8·5°, so that the experimental error ($\pm 0{\cdot}5°$) is not significant.

The hysteresis effect predicted by Hornung *et al.* (1979), that the unstable state of regular reflection in the range $a_N < \alpha < \alpha_d$ might be achieved by a continuous adjustment of the wedge incidence during the run, could not be confirmed. However, evidence of a hysteresis effect of the kind proposed is shown to be given by a particular unsteady experiment of Henderson & Lozzi (1979). The absence of hysteresis in our experiments suggests that the viscous growth of the shear layer downstream of the triple point affects the transition to an extent which is too small to be detected.

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